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We Make Stats Easy.

## Chapter 8

Tutorial Length  
1 Hour 30 Minutes

# Chapter 8 Note

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## Chapter Topics:

- Probability Density Function
- Uniform Distribution
- Normal Distribution

From this chapter, it is EXTREMELY important to learn how to use the Z-table and how to calculate probability using it. In this chapter we consider finding the probability for a single sample using the normal distribution. After going through the introductory examples and multiple choice questions, move forward to chapter 9. Only after chapter 8 and 9 are done should the past test questions be done. The reason being that in test 1, there are 2 short answer questions. The first one typically comes from chapter 7 and is usually a binomial question. The second short answer is a combination of material from chapter 8 and 9 (and once in a while chapter 10).

A copy of the Z-table for the normal distribution have been attached at the end of the file for reference.

In addition to this PDF, please download the formula sheet. This formula sheet is from the previous semester. It is recommended you check the course page for an updated posting on the formula sheet so that you can study with the most up to date version in case the one on this website is outdated.

# Probability Density Functions

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## Continuous Random Variables

A continuous random variable is a variable that can take on an uncountable number of values. In order to find probability for continuous random variables, we need to find the area under the probability density function,  $f(x)$ , in a given interval.

## Probability Density Functions

The probability density function,  $f(x)$ , is an equation or function that meet these two conditions

1.  $f(x) \geq 0$  for all  $x$  from  $a$  to  $b$  [That is, all values must be 0 or larger]
2. The total area under the curve must be equal to 1 from  $a$  to  $b$

Note 1: For continuous distributions,  $P(X = x) = 0$

Note 2: For continuous distributions,  $<$  and  $\leq$  mean the same thing, as does  $>$  and  $\geq$   
This is NOT true for discrete random variables!

## Note

Even though there is a difference between discrete and continuous random variables, when the countable number of values is large, we typically use a continuous probability distribution as an approximation. The most common continuous distribution is the Normal Distribution and it is often used for as a model for discrete random variables that take on a large number of values.

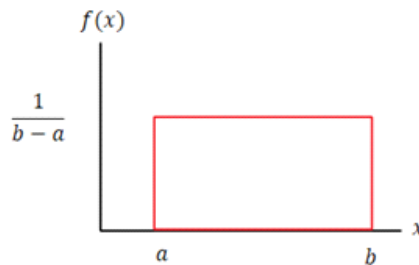
# Uniform Distribution

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## Uniform Probability Distribution

This distribution is also known as the rectangular probability distribution. The probability density function is a horizontal line from  $a$  to  $b$ .

$$f(x) = \frac{1}{b-a} \text{ for } a \leq x \leq b$$



In order to find the probability of any given scenario, we simply have to find the area of a rectangle using *base* x *height*.

$$P(x_1 < X < x_2) = (x_2 - x_1) * \frac{1}{b-a}$$

Note:  $x_1$  and  $x_2$  have to be within the interval from  $a$  to  $b$ .

# Uniform Distribution

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## Example

The amount of water used during a shower is uniformly distributed, with a minimum of 20 litres, and a maximum 250 litres.

A) What is the probability that the amount of water used is between 50 to 100 litres?

B) What is the probability that the amount of water used is less than 150 litres?

C) What is the probability that the amount of water used is at least 100 litres?

D) What is the probability that the amount of water used is greater than 300 litres?

E) What is the probability that exactly 200 litres will be used?

# Multiple Choice

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1. For the probability density function of a continuous random variable, which of the following is true?
  - a. The function has a countable number of possible values.
  - b. The probability at any one point is not zero.
  - c. The total area under the probability density function  $f(x)$  equals 1.
  - d. None of these statements are true.
2. For a continuous random variable, the probability density function,  $f(x)$  represents:
  - a. all possible values that  $X$  will assume within some interval  $a \leq x \leq b$ .
  - b. the height of the density function at  $x$ .
  - c. the probability that  $X$  takes on a specific value  $x$ .
  - d. All of the above
3. What is the difference between continuous and discrete random variables?
  - a. Continuous random variables assume an uncountable number of values, and discrete random variables do not.
  - b. The probability for any individual value of a continuous random variable is zero, but for discrete random variables it is not.
  - c. Probability for continuous random variables means finding the area under a curve, while for discrete random variables it means summing individual probabilities.
  - d. All of these choices are true.
4. Suppose  $f(x) = 0.20$ . What range of possible values can  $X$  take on and still have the density function be valid?
  - a.  $[0, 5]$
  - b.  $[2, 7]$
  - c.  $[-3, +2]$
  - d. All of these choices are true.
5. For a uniform random variable on the interval  $[a, b]$ , the shape of the probability density function is
  - a. A rectangle whose  $X$  values go from  $a$  to  $b$ .
  - b. A straight line with height  $(b - a)$  over the range  $[a, b]$ .
  - c. A discrete probability density function with the same value of  $f(x)$  from  $a$  to  $b$ .
  - d. All of these choices are false.
6. For a uniform random variable on the interval  $[a, b]$ , which statement is false?
  - a. The values of  $f(x)$  are different for various values of the random variable  $X$ .
  - b.  $f(x)$  constant for each possible value of  $X$ .
  - c.  $f(x)$  equals one divided by the length of the interval from  $a$  to  $b$ .
  - d. None of these choices.
7. Suppose  $f(x) = 1/4$  over the range  $a \leq x \leq b$ , and suppose  $P(X > 5) = 1/2$ . What are the values for  $a$  and  $b$ ?
  - a. 0 and 4
  - b. 3 and 7
  - c. Can be any range of  $x$  values whose length  $(b - a)$  equals 4.
  - d. Need more information to answer the question.

## Multiple Choice

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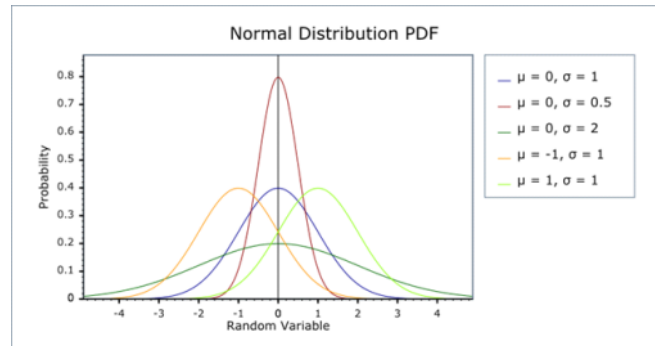
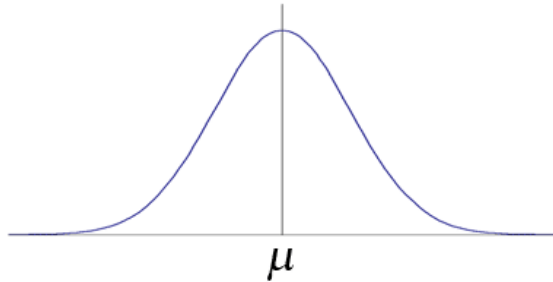
8. The probability density function  $f(x)$  for a uniform random variable on the interval  $[1, 9]$  is
- a. 0.20
  - b. 0.125
  - c. 8
  - d. None of these choices.
9. If the random variable  $X$  has a uniform distribution between 30 and 40, then  $P(25 \leq X \leq 35)$  is:
- a. 1.0
  - b. 0.5
  - c. 0.1
  - d. undefined
10. Which of the following does not represent a continuous uniform random variable?
- a.  $f(x) = 1/2$  for  $x$  between  $-1$  and  $1$ , inclusive.
  - b.  $f(x) = 10$  for  $x$  between  $0$  and  $1/10$ , inclusive.
  - c.  $f(x) = 1/4$  for  $x = 3, 4, 5, 6$ .
  - d. None of these choices represents a continuous uniform random variable.

# Normal Distribution

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## Normal Distribution

A normal distribution is symmetric and bell shaped. The mean, median, and mode are located in the center, and it is unimodal. To describe any normal distribution, you need to know only the mean  $\mu$  and standard deviation  $\sigma$ . The larger the standard deviation, the wider the curve, and vice versa. Probabilities can be calculated by finding the area under the curve. Since there are an infinite number of normal curves (many combinations of  $\mu$  and  $\sigma$ ), we make things easier by dealing with just one type of normal curve all the Standard Normal Curve.



Recall the Empirical Rule:

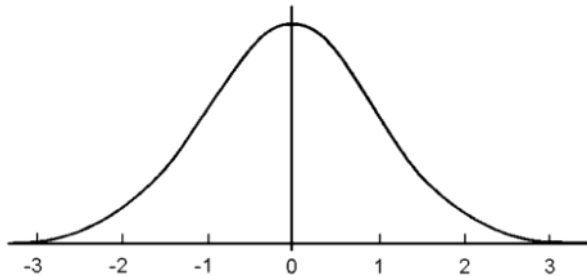
- 68% of all data lies within 1 standard deviation of the mean
- 95% of all data lies within 2 standard deviation of the mean
- 99.7% of all data lies within 3 standard deviation of the mean

The empirical rule applies only to bell-shaped distributions, and the normal curve is a bell-shaped distribution.

# Standard Normal Distribution

## Standard Normal Variable – Z

There is an infinite amount of normal curves that are possible by varying the mean and standard deviation. The **Standard Normal Variable** has a mean of 0 and a standard deviation of 1 and is represented by the letter Z.



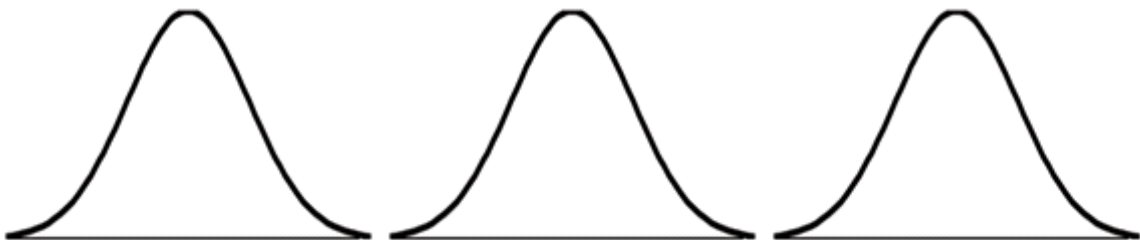
The probability density function for the standard normal variable (Z) is well defined and tables are used to calculate probabilities by finding the area under the Standard Normal curve.

### Notes

- A) The total area under the curve is 1. The area to the left is 0.5, and the area to the right 0.5
- B) The center of the curve is  $z = 0$  (since  $\mu = 0$  for the standard normal variable)
- C) Negative Z values on the left half of the graph, positive Z values on the right half.

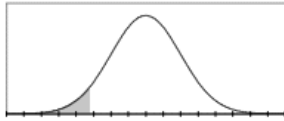
When trying to find probability using Z values, draw out the normal curve and shade in the area you are trying to find. When you look up the Z value from the Z table, keep in mind that the table entry ALWAYS represents area to the LEFT. In other words, the table entry represents cumulative probabilities  $P(Z \leq z_1)$ . You may have to manipulate that number in order to find the area you need to calculate.

Scenario	What you need	What to use from Z-table
	$P(Z = z_1)$	0
1	$P(Z < z_1)$ $P(Z \leq z_1)$	Table Entry
2	$P(Z > z_1)$ $P(Z \geq z_1)$	$1 - P(Z < z_1)$ $1 - P(Z \leq z_1)$
3	$P(z_1 < Z < z_2)$ $P(z_1 \leq Z \leq z_2)$ $P(z_1 \leq Z < z_2)$ $P(z_1 < Z \leq z_2)$	$P(Z < z_2) - P(Z < z_1)$ $P(Z \leq z_2) - P(Z \leq z_1)$ $P(Z < z_2) - P(Z \leq z_1)$ $P(Z \leq z_2) - P(Z < z_1)$



# Z Table - Left Side (Negative Z Values)

**Table 2 – Cumulative Standardized Normal Probabilities**

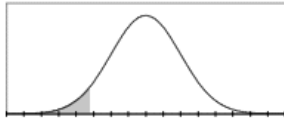


Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

LEFT SIDE VALUES,  $z < \mu$

# Z Table - Right Side (Positive Z Values)

**Table 2 – Cumulative Standardized Normal Probabilities**



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

RIGHT SIDE VALUES,  $z > \mu$

# Normal Distribution

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**Example** Determine the following probabilities.

$$P(Z < 1.25)$$

$$P(Z \leq 1.25)$$

$$P(Z < -0.63)$$

$$P(Z < -4.20)$$

$$P(Z > 1.48)$$

$$P(Z > 3.22)$$

$$P(-1.45 < Z < 2.21)$$

$$P(1.51 \leq Z < 1.98)$$

$$P(Z = -0.35)$$

# Normal Distribution

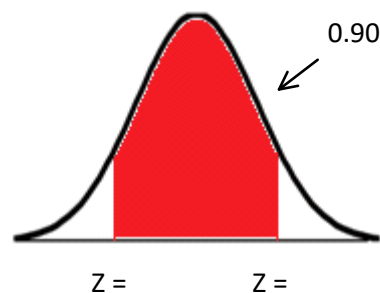
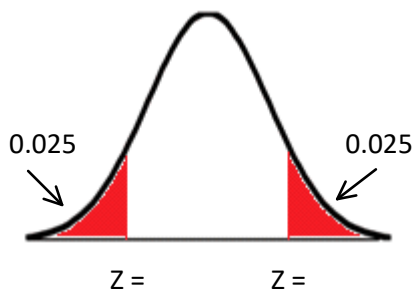
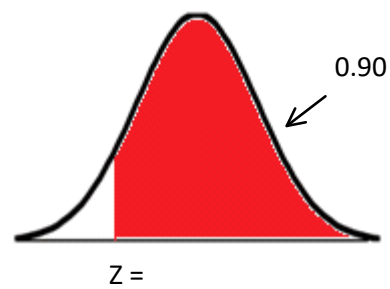
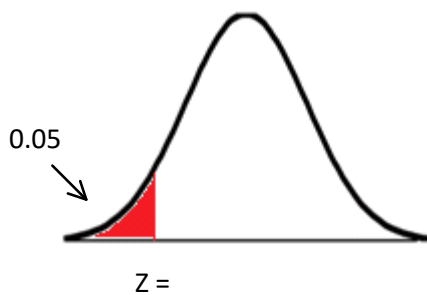
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There are times where you will know the probability (shaded area under the curve), and have to work backwards to determine what Z value represents that probability.

Step 1) Determine the area to the left, if not already given

Step 2) Look at the table entries and find the entry you are looking for. Once you find the entry, determine the corresponding Z value. If you can't find the exact entry, find the 2 closest entries and average their corresponding Z values.

**Example** Determine the Z value that corresponds to the shaded area.



# Normal Distribution

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## Application Questions

Typical application questions will give the mean and standard deviation and mention the data is normally distributed.  $X$  represents the raw data being measured (weight, height, volume, etc). The raw data has a mean of  $\mu$  and a standard deviation of  $\sigma$ . However, we need to convert this data to the Standard Normal Variable  $Z$ , where the mean is 0 and the standard deviation is 1.

## How to Convert $X$ into $Z$

$$Z = \frac{X - \mu}{\sigma}$$

The  $Z$  value (frequently called the  $Z$  score) represents the number of standard deviations a given  $X$  score is above or below the mean.  $Z$  scores can also be used to compare different data sets. The higher the  $z$  score, the higher the percentile.

## Question Type 1 – Finding a probability given a raw score

$X \rightarrow Z \rightarrow Probability$

1. Draw a diagram, label the mean in the center, and shade the region being sought out
2. Write down what the question is asking for in  $P$  notation
3. Using the raw data value,  $X$ , convert to a  $Z$ -value using  $Z = \frac{X - \mu}{\sigma}$
4. Rewrite the  $P$  notation in terms of  $Z$
5. Use the  $Z$  table to answer the question

## Question Type 2 – Finding a raw score given a probability

$Probability \rightarrow Z \rightarrow X$

1. Draw a diagram, label the mean in the center, and shade the area the question mentions
2. Determine the total area to the left
3. Find the  $z$ -value in the table that corresponds to that area to the left
4. Convert the  $z$ -value to a raw score using  $X = Z\sigma + \mu$

# Normal Distribution

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## Example

The duration of television commercials is normally distributed with a mean of 75 seconds and a standard deviation of 20 seconds. What is the probability that a commercial will last less than 35 seconds?

## Example

Suppose the number of bacteria in 1 millilitre [ml] of drinking water is normally distributed, with a mean of 85 and a standard deviation of 9. What is the probability that a 1 ml sample will contain more than 100 bacteria?

# Normal Distribution

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## Example

Assume that the mean age of farm workers in Canada is 45 years with a standard deviation of 5 years. What is the probability that a worker selected at random would be more than 50 years old.

## Example

The value of an item can be defined as the amount that someone is willing to pay for it. 1000 people were asked to name the maximum price they would pay for a particular painting. The data was normally distributed with a mean of \$225.00 and a standard deviation of \$45.00. What is the maximum price that a person in the bottom 10 percent of the group was willing to pay ?

# Multiple Choice

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11. Which of the following is false regarding a normal distribution?
- It is symmetrical.
  - The mean is always zero.
  - The mean, median, and mode are all equal.
  - It is a bell-shaped distribution.
12. If  $X$  has a normal distribution with mean 60 and standard deviation 6, which value of  $X$  corresponds with the value  $z = 1.96$ ?
- $x = 71.76$
  - $x = 67.96$
  - $x = 61.96$
  - $x = 48.24$
13. A standard normal distribution is a normal distribution with:
- a mean of zero and a standard deviation of one.
  - a mean of one and a standard deviation of zero.
  - a mean always larger than the standard deviation.
  - None of these choices.
14. Approximately what proportion of the data from a normal distribution is within two standard deviations from the mean?
- 0.3413
  - 0.4772
  - 0.6826
  - 0.9544
15. If  $Z$  is a standard normal random variable, the area to the left of a value  $z$  is expressed as
- $P(Z \geq z)$
  - $P(Z \leq z)$
  - $P(0 \leq Z \leq z)$
  - $P(Z \geq -z)$
16. Given that  $Z$  is a standard normal variable, the variance of  $Z$ :
- is always greater than 2.0.
  - is always greater than 1.0.
  - is always equal to 1.0.
  - cannot assume a specific value.
17. If  $Z$  is a standard normal random variable, a negative value ( $z$ ) on its distribution would mean
- $z$  is to the left of the mean.
  - the standard deviation of this  $Z$  distribution is negative.
  - the area between zero and the value  $z$  is negative.
  - None of these choices.

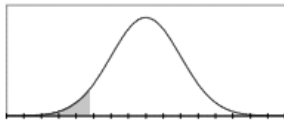
# Multiple Choice

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18. A larger standard deviation of a normal distribution indicates that the distribution becomes:
- narrower and more peaked.
  - flatter and wider.
  - more skewed to the right.
  - more skewed to the left.
19. In its standardized form, the normal distribution:
- has a mean of 0 and a standard deviation of 1.
  - has a mean of 1 and a variance of 0.
  - has an area equal to 0.5.
  - cannot be used to approximate discrete probability distributions.
20. Most values of a standard normal distribution lie between:
- 0 and 1
  - 3 and 3
  - 0 and 3
  - $-\infty$  to  $\infty$
21. John took a math test whose mean was 70 and standard deviation was 5. The total points possible was 100. John's results were reported to be at the 95th percentile. What was John's actual exam score, rounded to the nearest whole number?
- 95
  - 78
  - 75
  - 62
22. John took a statistics test whose mean was 80 and standard deviation was 5. The total points possible was 100. John's score was 2 standard deviations below the mean. What was John's score, rounded to the nearest whole number?
- 78
  - 70
  - 90
  - None of these choices.
23. Janice took a psychology exam whose mean was 70 with standard deviation 5. She also took a calculus exam whose mean was 80 with standard deviation 10. She scored 85 on both exams. On which exam did she do better compared to the other students who took the exam?
- She did better on the psychology exam, comparatively speaking.
  - She did better on the calculus exam, comparatively speaking.
  - She did the same on both exams, relatively speaking.
  - Cannot tell without more information.
24. Suppose Megan's exam score was at the 75th percentile on an exam whose mean was 80. What was Megan's exam score?
- 76.81
  - 72.00
  - 80.00
  - Cannot tell without more information.

# Z Table - Left Side (Negative Z Values)

**Table 2 – Cumulative Standardized Normal Probabilities**

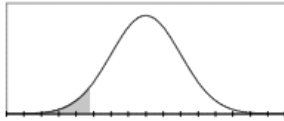


Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

LEFT SIDE VALUES,  $z < \mu$

# Z Table - Right Side (Positive Z Values)

**Table 2 – Cumulative Standardized Normal Probabilities**



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

RIGHT SIDE VALUES,  $z > \mu$