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## Chapter 11

Tutorial Length  
1 Hour 45 Mins

# Chapter 11 Note

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In this chapter we get an introduction to hypothesis testing. Each chapter going forward, until the end of the course, will introduce new methods of hypothesis testing for different scenarios. It is important to learn the basics of hypothesis testing in this section in order to progress into the other chapters.

Chapter Topics:

- Hypothesis testing
- Testing mean of a single population when  $\sigma$  is known
- Hypothesis testing errors (Type I and II)
- Calculating Probability of Type II Error ( $\beta$ )

Please download the exam formula sheet. This formula sheet is from the previous semester. It is recommended you check the course page for an updated posting on the formula sheet so that you can study with the most up to date version in case the one on this website is outdated.

## How to use ADMS2320.com

The goal of this website is to quickly and efficiently make you understand all the **major concepts**. We have many years of experience tutoring students for ADMS 2320 and we have a pretty good idea of what you need to concentrate on in order to do well.

Each chapter, we will go through theory and then apply that theory through multiple choice and short answer questions. We attempt to make our videos quick and to the point. As such, it is a good idea to always review your class notes and textbook readings to go over additional topics and examples.

**Note:** Not all theory is covered prior to answering practice questions. Some theory is taught while answering questions in order to put the theory into immediate practice. Make sure to go through all the videos in a given chapter.

# Hypothesis Testing

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## Hypothesis Testing

**Hypothesis** – A statement regarding population parameter(s)

Null Hypothesis ( $H_0$ )– In general, the null hypothesis is the value we expect the parameter to be equal to.

Alternative Hypothesis ( $H_1$ ) – Stating the population parameter is **not equal** the given one, or is **greater than** the given one, or **less than** the given one.

All hypothesis testing questions involve testing whether the null hypothesis should be rejected or not rejected. If the null is rejected, then the statistical evidence supports the alternative. If the null is not rejected, there is not enough evidence to support the alternative. Hypothesis testing doesn't tell us which hypothesis is "true", just which hypothesis is likely based on probability.

**Significance Level  $\alpha$**

Typically 5% (Usually ranges 1-10%)

**Confidence Level  $1 - \alpha$**

Typically 95%. (Usually ranges 90-99%)

Both significance level and confidence level's are between 0 to 100% inclusive (0 to 1 in decimal form)

**Statistically Significant Result**

A result that is not due to chance. (There is a low probability of it being a random occurrence). In statistics, a 'significant' result means the null hypothesis is rejected.

"The result of this experiment is significant at 0.05 significance level."

This means the null hypothesis has been rejected as there is a less than 5% probability of getting this value from sampling error.

# Hypothesis Testing

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## Hypothesis Testing

The purpose of hypothesis testing is to see if there is enough evidence to determine if a given population parameter is other than what we already expect it to be. There are two hypotheses, the null and alternative. We assume the null hypothesis is true and through the testing procedure we can determine if there is evidence to support the alternative hypothesis. In general, researchers are interested in seeing if there is evidence to support or not support the alternative hypothesis.

- **Step 1 – State the null and alternative hypothesis**

The null hypothesis ( $H_0$ ) contains an = symbol. Ex:  $H_0: \mu = 25$

The alternative hypothesis ( $H_1$ ) contains one of the following:  $<, \neq, >$  Ex:  $H_1: \mu \neq 25$

Note: Some hypothesis testing will not have any symbols, just sentences.

- **Step 2 – Calculate test statistic**

This will be a formula unique to the situation being tested.

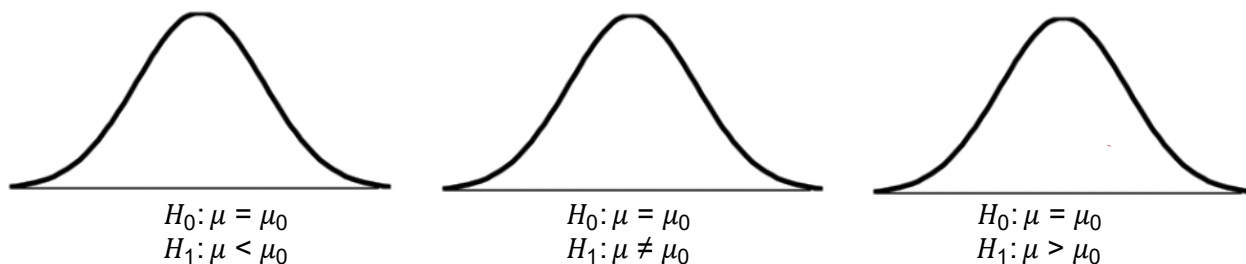
- **Step 3 – Determine significance level  $\alpha$  (0.05 if not stated) and rejection region**

If the alternative hypothesis has the  $<$  symbol, it is a LEFT tailed test

If the alternative hypothesis has the  $\neq$  symbol, it is a TWO tailed test

If the alternative hypothesis has the  $>$  symbol, it is a RIGHT tailed test

In some situations, there are no symbols in the alternate hypothesis. For these hypothesis testing scenarios, the required rejection region will be pointed out when the formula is introduced.



- **Step 4 – Two Part conclusion:**

Part 1: Reject or do not reject  $H_0$

Reject if the test value lies in the rejection region found in step 3, otherwise do not reject. Alternatively, reject if the P-value (probability) is less than  $\alpha$  for the test statistic found in step 2. Note: If it's a 2 tailed test, make sure to multiply P by 2 before comparing to  $\alpha$ . For 2 tailed tests, confidence intervals can also be used to make a rejection determination.

Part 2: Concluding statement supporting/not supporting alternate hypothesis

If null hypothesis is rejected, there is evidence to support the alternative hypothesis.  
If null hypothesis is not rejected, there is not enough evidence to support the alternative.  
Notice how the focus is always on the alternate in terms of a conclusion.

"Reject  $H_0$ , at  $\alpha = ?$ , there is evidence to support...."

"Do not reject  $H_0$ , at  $\alpha = ?$ , there is not enough evidence to support... "

# Population Mean, $\sigma$ is known, 1 Population

## Inference About A Population Mean When The Standard Deviation Is Known

When to use: describing a single population with interval data types and the interest is central location.

### Hypothesis

$$\begin{aligned} H_0: \mu &= \mu_0 \\ H_1: \mu &< \mu_0 \text{ OR} \\ &\mu \neq \mu_0 \text{ OR} \\ &\mu > \mu_0 \end{aligned}$$

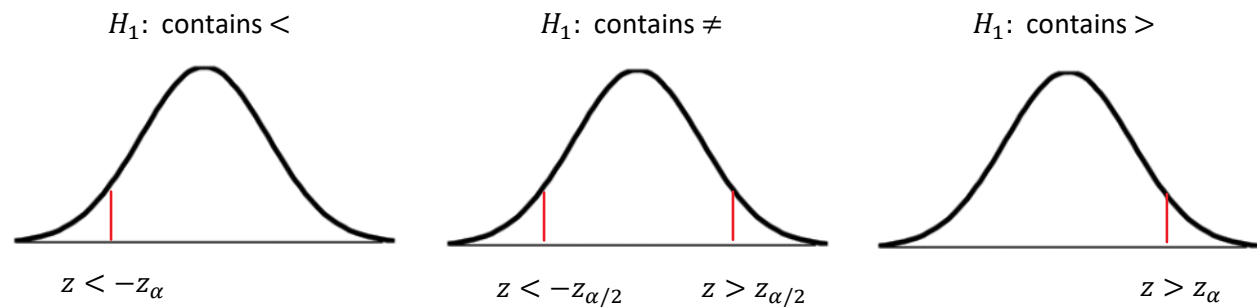
### Test Statistic

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

### Confidence Interval Estimator

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

## Rejection Regions For Z Distribution



### Common Values

$z_{0.05} = 1.645$
$z_{0.025} = 1.96$
$z_{0.01} = 2.33$
$z_{0.005} = 2.575$

## Rejection Region Method vs P-Value Method vs Confidence Interval Method

Rejection region method is a simple reject/do no reject without further information.

P-value method gives a probability as to how likely observing a sample statistic at least as extreme as the one found is, assuming the null hypothesis is true. P-value is NOT the probability that the null hypothesis is true.

Confidence interval rejection method only gives a simple reject/do not reject similar to the rejection region method. It is also unable to provide a P-value. Furthermore, a confidence interval method is usually only used for 2-tail tests (it's possible to do for 1-tail but more work is involved).

Remember, in general, "estimate" means find the confidence interval, and "determine" or "can it be concluded" or similar means hypothesis testing.

# Population Mean, $\sigma$ is known, 1 Population

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## Example:

A researcher reports that the average salary of sales managers is \$42,000. A sample of 30 sales managers has a mean salary of \$43,250. Assume the standard deviation of the population is \$5250.

A) Perform a hypothesis test to test the claim that sales managers earn more than \$42,000 per year. (For the sake of practice, we will do this using the rejection region method and the p-value method.)

B) Estimate the true population mean

## Population Mean, $\sigma$ is known, 1 Population

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### Example:

HigherED magazine estimates the average cost of tuition is \$5,500 in Canada. To see if the average cost is different for York University students, a survey was conducted with 25 students and the average cost of tuition was found to be \$5750. Assume the population standard deviation for tuition is \$550.

A) Can it be concluded that the average tuition price for York students is different. (For the sake of practice, we will do this using the rejection region method and the p-value method)

B) Find the 95% confidence interval for the amount of tuition all York students pay. Determine if this interval supports your answer from part A.

## Population Mean, $\sigma$ is known, 1 Population

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### Example:

Professors teaching introductory calculus recommend spending 4 hours a week doing homework problems in order to do well in the class. A semester lasts 12 weeks long so over the course of the semester students should have spent 48 hours doing homework. At the end of the semester, a calculus professor chose a random sample of 10 students and asked them how many hours they had spent in total doing homework problems. The responses from the student are listed below. Assuming  $\sigma^2 = 100$ , determine if there is evidence that the average student spent less than the recommended time on homework. Use a 10% significance level.

55 45 27 35 39 42 45 29 35 49



# Multiple Choice

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1. A sales rep at a shoe store does not believe the claim that the average female over 18 has 7 pairs of shoes. He believes that numbers to be higher. What would be the null and alternative hypothesis?

- a.  $H_0: \mu = 7$  vs.  $H_1: \mu \neq 7$
- b.  $H_0: \mu \neq 7$  vs.  $H_1: \mu = 3$
- c.  $H_0: \mu = 7$  vs.  $H_1: \mu > 7$
- d.  $H_0: \mu = 7$  vs.  $H_1: \mu < 7$

2. The researcher is usually interested in which hypothesis?

- a. Null Hypothesis
- b. Alternative Hypothesis
- c. Both Null and Alternative
- d. Neither

3. A researcher believes the average number of t-shirts a person owns is greater than 9. What are the null and alternative hypothesis?

- a.  $H_0: \mu = 9$  vs.  $H_1: \mu < 9$
- b.  $H_0: \mu = 9$  vs.  $H_1: \mu > 9$
- c.  $H_0: \mu > 9$  vs.  $H_1: \mu = 9$
- d.  $H_0: \mu < 9$  vs.  $H_1: \mu = 9$

4. Which of the following is a possible null hypothesis?

- a.  $\mu = 5$
- b.  $\bar{x} = 5$
- c.  $\bar{\mu} \neq 5$
- d. All of these are possible options.

5. If a researcher wants to determine the average monthly cell phone bill is less than \$65, what kind of hypothesis test should they perform?

- a. One-tail test
- b. Two tail test
- c. Both tests would provide the same result
- d. All of the above

6. For a two-tail test with 5% level of significance, the null hypothesis will be rejected if the z test statistic is:

- a. less than  $-1.96$  or greater than  $1.96$
- b. less than  $1.96$  or greater than  $-1.96$
- c. greater than  $-1.96$  or smaller than  $1.96$
- d. None of the above

7. Consider the following  $H_0: \mu = 25$  vs.  $H_1: \mu \neq 25$ ,  $n = 36$ ,  $\bar{x} = 25.9$ , and  $\sigma = 9$ . What does the z test statistic equal to?

- a. 0.6
- b.  $-0.6$
- c. 0.1
- d.  $-0.1$

# Multiple Choice

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8. At a significance level of  $\alpha = 0.1$ , if the hypothesis is not rejected, then the hypothesis...
- a. will not be rejected at the 0.05 level.
  - b. must be rejected at the 0.05 level.
  - c. must be rejected at the 0.025 level.
  - d. may be rejected at the 0.05 level.
9. At  $\alpha = 0.05$ , which of the following p-values would lead to a rejection of the null hypothesis?
- a. 0.20
  - b. 0.001
  - c. 0.055
  - d. 0.995
10. If a null hypothesis is rejected at the 0.1 level and 0.05 level, but not at the 0.01 level, then what is the most accurate statement regarding the p-value?
- a.  $0.01 < \text{p-value} < 0.05$ .
  - b.  $0.05 < \text{p-value} < 0.01$ .
  - c.  $\text{p-value} = 0.05$ .
  - d.  $\text{p-value} = 0.10$ .
11. Consider the following hypothesis test:  $H_0: \mu = 165$  vs.  $H_1: \mu > 165$ . If the z test statistic is equal to 1.76, then the calculated the p-value is:
- a. 0.9608
  - b. 0.0392
  - c. 0.0784
  - d. Cannot determine without significance value
12. Consider the following hypothesis test:  $H_0: \mu = 165$  vs.  $H_1: \mu \neq 165$ . If the z test statistic is equal to 1.76, then the calculated the p-value is:
- a. 0.9608
  - b. 0.0392
  - c. 0.0784
  - d. None of the above
13. Consider the following hypothesis setup:  $H_0: \mu = 50$  vs.  $H_1: \mu > 50$ . A sample mean of 52.5 was used to calculate a test statistic which has a corresponding p-value of 0.0251. Which statement best represents what this p-value means?
- a. The likelihood of observing a sample mean of 52.5 from a population whose mean is 50 is 0.0251.
  - b. The probability of observing a sample mean at least as large as 52.5 from a population whose mean is 50 is 0.0251.
  - c. The probability that the population mean is larger than 50 is 0.0251.
  - d. None of these choices.

# Hypothesis Testing Errors

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## Hypothesis Testing Errors

### Type I Error - "False Positive"

- Rejecting the null hypothesis when it is actually true
- Probability of type I error is equivalent to the significance level  $\alpha$

### Type II Error - "False Negative"

- Accepting (Not rejecting) the null hypothesis when it is actually false
- Probability of type II error is referred to as  $\beta$

As the likelihood of one error increases, the other decreases, assuming sample size stays the same. To reduce the chance of a Type II error ( $\beta$ ), you can increase  $\alpha$  but that increases the chance of a type I error. Instead, increase the sample size. This will help to reduce  $\beta$ .

## Power of a Test

The probability of rejecting null hypothesis when it is false. Power =  $1 - \beta$

## Example

Consider a person on trial for a crime. The status quo is we must assume innocent.

$H_0$ : The defendant is innocent

$H_1$ : The defendant is not innocent

	$H_0$ True (Person is innocent)	$H_0$ False (Person is not innocent)
Reject $H_0$ (Jury Convicts)	Type 1 Error	Correct Decision
Do not reject $H_0$ (Jury Acquits)	Correct Decision	Type II Error

## Hypothesis Testing Error Chart

	$H_0$ True	$H_0$ False
Reject $H_0$	Type 1 Error ( $\alpha$ )	Correct Decision ( $1 - \beta$ )
Do not reject $H_0$	Correct Decision	Type II Error ( $\beta$ )

# Multiple Choice

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14. At a given level of significance  $\alpha$ , by increasing sample size the probability of a Type I error will \_\_\_\_\_ and the probability of a Type II error will \_\_\_\_\_
- a. increase, decrease.
  - b. decrease, stay the same
  - c. stay the same, increase.
  - d. stay the same, decrease.
15. For a fixed sample size, increasing the level of significance  $\alpha$  will
- a. decrease probability of a type II error.
  - b. increase probability of a type II error.
  - c. not change the probability of a type II error.
  - d. Decrease the probability of a type I error.
16. The likelihood of a Type I error is:
- a.  $\alpha$
  - b.  $\beta$
  - c.  $1 - \alpha$
  - d. None of the above
17. Which of the following describes a type I error?
- a. Making a correct decision when the null hypothesis is false.
  - b. Making an incorrect decision when the null hypothesis is false.
  - c. Making an incorrect decision when the null hypothesis is true.
  - d. Making a correct decision when the null hypothesis is true.
18. What situation results in a Type I error?
- a. rejecting a false null hypothesis.
  - b. not rejecting a false null hypothesis.
  - c. rejecting a true null hypothesis.
  - d. not rejecting a true null hypothesis.
19. A type II error cannot occur when
- a. null hypothesis is false.
  - b. null hypothesis is true.
  - c. don't reject the null hypothesis
  - d. None of the above

# Multiple Choice

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20. Which of the following statements are true?

- a. As the probability of making a Type I error decreases, the probability of a Type II error increases.
- b. The level of significance and the probability of making a Type I error are the same
- c. The power of the test decreases as the level of significance decreases.
- d. All of these choices are true.

21. A Type I error probability of  $\alpha = 0.05$  means that:

- a. We don't reject a true null hypothesis 5% of the time.
- b. We reject a true null hypothesis 5% of the time
- c. We reject a false null hypothesis 5% of the time.
- d. None of the above

22. Consider the following hypothesis,  $H_0: \mu = 20$  vs.  $H_1: \mu > 20$ . If the actual mean is 20 but our hypothesis testing leads us to conclude it is greater than 20, what has occurred?

- a. A Type I error.
- b. A Type II error.
- c. The correct conclusion.
- d. None of the above.

# Type II Error

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## Recall

A type II error is not rejecting the null hypothesis when it is actually false. The probability of a type II error is  $\beta$ . Power is the probability of rejecting null hypothesis when it is false.

Power =  $1 - \beta$

## Type II Error ( $\beta$ ) Calculations

Step 1)

Find the rejection region critical value in terms of  $\bar{x}$  using the  $\mu$  in the **null** hypothesis

- Use  $-z_\alpha$  (left tail test)
- Use  $z_\alpha$  (right tail test)
- Use  $\pm z_{\alpha/2}$  (2 tail test)

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \rightarrow \bar{x} = z \frac{\sigma}{\sqrt{n}} + \mu$$

Step 2)

Calculate the Z test statistic using the  $\bar{x}$  values from step 1 and the **TRUE**  $\mu$

- For a 1 tail test, there will be 1 z-value
- For a 2 tail test, there will be 2 z-values

Using the test statistic, find the probability of NOT rejecting.

- If the hypothesis test was a left tail test, find the probability to the right of the z-value.
- If the hypothesis test was a right tail test, find the probability to the left of the z-value.
- If the hypothesis test was a 2-tail test, find the probability between the two z-values.

This probability is equal to  $\beta$ .

If needed, Power =  $1 - \beta$ .

# Type II Error

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## Example:

Find the probability of a Type II error ( $\beta$ ) for the following hypothesis test given that  $\mu = 52$ .

$$H_0: \mu = 50$$

$$H_1: \mu > 50$$

$$\alpha = 0.05, \sigma = 10, n = 50$$

## Example:

Find the probability of a Type II error ( $\beta$ ) for the following hypothesis test given that  $\mu = 104$ .

$$H_0: \mu = 100$$

$$H_1: \mu \neq 100$$

$$\alpha = 0.05, \sigma = 5, n = 25$$

# Type II Error

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**Example:**

Compute the power of the test if  $\mu = 26$ .

$$H_0: \mu = 30$$

$$H_1: \mu < 30$$

$$\alpha = 0.05, \sigma = 100, n = 100$$