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We Make Stats Easy.

Chapter 10

Tutorial Length
40 Minutes

Chapter 10 Note

Chapter Topics:

- Estimation
- Confidence Intervals
- Sample Size To Estimate The Mean

A copy of the Z-table for the normal distribution have been attached at the end of the file for reference.

In addition to this PDF, please download the formula sheet. This formula sheet is from the previous semester. It is recommended you check the course page for an updated posting on the formula sheet so that you can study with the most up to date version in case the one on this website is outdated.

Estimation

Point Estimator – Estimating an unknown population parameter with a single number. For example, using the sample mean $\bar{x}$ as an estimate for the population mean μ.

Cons to using a point estimator
-Highly probable the estimate is wrong ($\bar{x} \neq \mu$ in most cases)
-We can't tell how close the point estimate is to the actual parameter
-Can't tell how much it is affected by sample size

Interval Estimator – Estimating the unknown population parameter using an interval
-Takes into account sample size
-Gives a range for the population parameter along with a reasonable likelihood that the population parameter belongs in that range.

Properties of good estimators

1. Unbiased Estimator – Estimator whose *expected* value is equal to the population parameter. In statistics, we want our estimators to be unbiased!

   $\bar{x}$ (sample mean) is an unbiased estimator of μ (population mean)
   s^2 is an unbiased estimator of σ^2

2. Consistency – As sample size grows, the sample statistic should be closer to population parameter. Therefore the sampling error should be smaller as sample size grows.

3. Relative Efficiency – Given two unbiased estimators for a parameter, the estimator with the smaller variance is considered to have relative efficiency

   The sample mean and sample median are both unbiased estimators for the population mean, but the sample mean has less variance so it has relative efficiency.

Confidence Intervals

Significance Level
α is called the significance level (typical range is 0.5% - 10%)
When a significance level is not specified in a question, we assume α = 0.05 (5%)

Confidence Level
$1 - \alpha$ is called the confidence level (typical range is 90-99%)

Confidence Intervals
Estimating Population Mean When Population Standard Deviation/Variance is Known

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

LCL – Lower confidence limit $\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$
UCL – Upper confidence limit $\bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

Confidence interval is both numbers written in the following format: (LCL, UCL)

Common Confidence Levels and $z_{\alpha/2}$ Values

Confidence Level $1 - \alpha$	Significance Level α	$\alpha/2$	$z_{\alpha/2}$
0.9 (90%)	0.10 (10%)	0.05	$z_{0.05}$ = 1.645
0.95 (95%)	0.05 (5%)	0.025	$z_{0.025}$ = 1.96
0.98 (98%)	0.02 (2%)	0.01	$z_{0.01}$ = 2.33
0.99 (99%)	0.01 (1%)	0.005	$z_{0.005}$ = 2.575

What does a confidence interval mean?

If you were to perform an experiment over and over again by sampling the population and selecting the same sample size each time, from all the confidence intervals created, $(1 - \alpha)$ percent of the intervals would contain the true population mean. Another way to think of it, the estimate is only correct $1 - \alpha$ percent of the time.

The confidence interval DOES NOT mean there is a $(1 - \alpha)$ percent chance that the interval contains the population parameter of interest, in this case μ.

Example
A statistics student is assigned a term project to determine how long it takes for students to sell their textbooks to other students at the end of a semester. A sample of 50 people were asked how long it took them to sell their book and the average was 16.2 days. Assuming the population standard deviation is 2.4 days, find the following:

1. A point estimate for the mean amount of time to sell the books
2. A 95% confidence interval for the population mean. Explain what this interval means.

Example
The amount of snow fall in New York City was recorded over a 30 day period during the winter season. The average snow fall was 12cm with the population standard deviation being 1.5cm. Estimate the true mean population of the amount of snow fall during the winter season with 90% confidence.

Notes on the widths of confidence intervals and their widths.

There are four variables in the confidence interval formula. We usually want a narrow confidence interval instead of a wide interval. This allows us to have a better idea of what the population parameter is. Let's discuss what happens to the interval as different terms are manipulated.

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

As mean increases/decreases, nothing happens to the width.

As confidence level increases, width increases
As confidence level decreases, width decreases
Note: Usually we want at least a 95% interval unless otherwise specified.

As sample size increases, width decreases.
As sample size decreases, width increases.

As standard deviation/variance increases, width increases.
As standard deviation/variance decreases, width decreases.
Note: we can't control standard deviation/variance of a population, it is what it is.

Error of Estimation (Margin of Error)

$$B = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

B represents the margin of error, otherwise known as the bound on the error of estimation.

Recall confidence interval formula: $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$
B is just the second part of the formula.

Sample Size To Estimate A Population Mean

If we know what we want our margin of error to be (B), we can figure out what sample size to use in order to create a confidence interval, as long as we know what confidence level we want and what the standard deviation or variance is as well.

$n = \left(\frac{z_{\alpha/2}\,\sigma}{B}\right)^2$ Note: Always round your sample size up to the nearest whole number!

Example

A ice cream shop owner wants to determine the 95% confidence interval of the true mean cost of an ice cream cone with 2 scoops. How large should the sample be if he wants to be accurate to within $0.05? Historically, the standard deviation in price has been $0.15.

Multiple Choice

1. Which of the following is a characteristic for a good estimator?

   a. Being biased
   b. Being consistent
   c. Having relative inefficiency
   d. Both B & C are true.

2. If there are two unbiased estimators of a population parameter available, the one that has the smallest variance is said to be:

   a. an unbiased estimator.
   b. relatively consistent
   c. relatively efficient.
   d. consistent

3. A professor teaches a very large class of students and wants to determine the number of students that attend her lecture on a daily basis She asks a student to count the number of students that attend her class each day for the next 5 days. The student finds the average attendance to be 340 students. The student also provides an interval estimate of between 290 to 390 students come to class each day. What is an efficient, unbiased point estimate for the number of students that attend class each day?

   a. 290
   b. 340
   c. 390
   d. All of these choices.

4. If the confidence level is increased, the confidence interval:

   a. widens.
   b. narrows.
   c. remains the same.
   d. None of the above

5. Regarding a 90% confidence interval, which of the following statements is true?

   a. By repeatedly drawing samples of the same size from the same population, 90% of the resulting confidence intervals will include μ.
   b. There is a 90% probability that the population mean μ will lie between the lower confidence limit (LCL) and the upper confidence limit (UCL).
   c. We are 90% confident that our sample mean equals the population mean μ
   d. None of the above are true

6. The width of a confidence interval estimate of the population mean decreases when the:

   a. value of the population standard deviation decreases
   b. sample size increases
   c. level of confidence decreases
   d. All of these choices are true.

7. If the confidence interval estimate is determined to be 50.62 to 78.28 when a sample of 50 observations is used with a population standard deviation σ of 2.5, what is the mean of the sample?

a.64.45
b.23.52
c.69.62
d. None of the above

8. A confidence interval of 45.2 ± 3.03 was developed. The population standard deviation was assumed to be σ = 5. If σ was instead σ = 10, what would be the new confidence interval?

a. 50.2 ± 3.03
b. 50.2 ± 8.03
c. 45.2 ± 6.06
d. None of the above

9. If a confidence interval is too wide, how can the interval be made smaller?

a. Increase population standard deviation σ
b. Increase the sample size n
c. Increase confidence level
d. All of these choices.

10. Given a 90% confidence interval for the population mean μ is (512, 620). If you are 90% confident, this means the population mean μ would be in the resulting interval

a. 90% of the time during repeated sampling.
b. 90% of the observations in the population would be in the given interval.
c. 90% of the observations in the sample would be in the given interval.
d. All of these choices.

11. You wish to estimate the total salary paid to professors at public universities. Data from the government was used to gather data on 25 randomly selected professors. The 90% confidence interval for average salary was calculated to be ($78,000, $156,000). Based on this interval, do you believe the actual average salary for professors could be $105,250?

a. Yes, with 95% certainty
b. No, with 90% certainty
c. Yes, for sure.
d. Yes, with 90% certainty

12. You wish to estimate with 95% confidence the mean of a normal population whose standard deviation is 3. If you want the maximum error to be within ± 0.5 units, what should be your minimum sample size?

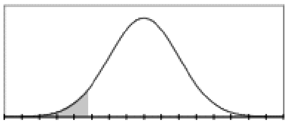
a. 138
b. 139
c. 150
d. None of these choices.

13. The estimated population mean within ± 3 units was calculated to be 65 when using 92 samples. What confidence level was used if the population standard deviation is 12.3?

a. 90%
b. 95%
c. 98%
d. 99%?

Z Table - Left Side (Negative Z Values)

Table 2 – Cumulative Standardized Normal Probabilities

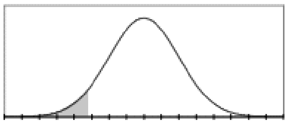


LEFT SIDE VALUES, $z < \mu$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Z Table - Right Side (Positive Z Values)

Table 2 – Cumulative Standardized Normal Probabilities



RIGHT SIDE VALUES, $z > \mu$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990